30 *Years*

Previous Solved Papers

GATE 2024

Computer Science & Information Technology

- Fully solved with explanations
- Analysis of previous papers
- Topicwise presentation
- Thoroughly revised & updated

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GATE - 2024

Computer Science & IT

Topicwise Previous GATE Solved Papers (1994-2023)

Editions

1st Edition	:	2007
2nd Edition	:	2008
3rd Edition	:	2009
4th Edition	:	2010
5th Edition	:	2011
6th Edition	:	2012
7th Edition	:	2013
8th Edition	:	2014
9th Edition	:	2015
10th Edition	:	2016
11th Edition	:	2017
12th Edition	:	2018
13th Edition	:	2019
14th Edition	:	2020
15th Edition	:	2021
16th Edition	:	2022
17th Edition	**:**	**2023**

Preface


B. Singh (Ex. IES)

Over the period of time the GATE examination has become more challenging due to increasing number of candidates. Though every candidate has ability to succeed but competitive environment, in-depth knowledge, quality guidance and good source of study is required to achieve high level goals.

The new edition of **GATE 2024 Solved Papers : Computer Science & Information Technology** has been fully revised, updated and edited. The whole book has been divided into topicwise sections.

At the beginning of each subject, analysis of previous papers are given to improve the understanding of subject.

I have true desire to serve student community by way of providing good source of study and quality guidance. I hope this book will be proved an important tool to succeed in GATE examination. Any suggestions from the readers for the improvement of this book are most welcome.

B. Singh (Ex. IES)
Chairman and Managing Director
MADE EASY Group

GATE-2024

Computer Science & IT

CONTENTS

Discrete & Engineering Mathematics

UNIT

I

CONTENTS

Discrete & Engineering Mathematics

Syllabus

Mathematical Logic: Propositional and first order logic.

Set Theory & Algebra: Sets, relations, functions, partial orders and lattices. Groups.

Combinatorics: Counting, recurrence relations, generating functions.

Graph Theory: Connectivity, matching, coloring.

Probability: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Analysis of Previous GATE Papers

Exam Year	1 Mark Ques.	2 Marks Ques.	5 Marks Ques.	Total Marks
1994	4	5	–	**14**
1995	4	5	1	**19**
1996	7	7	–	**21**
1997	4	7	–	**18**
1998	7	7	1	**26**
1999	4	5	2	**24**
2000	2	5	2	**22**
2001	4	5	–	**14**
2002	6	4	3	**29**
2003	5	15	–	**35**
2004	5	11	–	**27**
2005	5	10	–	**25**
2006	3	10	–	**23**
2007	4	9	–	**22**
2008	4	10	–	**24**
2009	4	6	–	**16**
2010	6	8	–	**24**
2011	–	5	–	**10**

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2012	6	5	**16**
2013	6	3	**12**
2014 Set-1	6	9	**24**
2014 Set-2	5	8	**21**
2014 Set-3	7	8	**23**
2015 Set-1	5	8	**21**
2015 Set-2	5	8	**21**
2015 Set-3	7	6	**19**
2016 Set-1	5	4	**13**
2016 Set-2	6	3	**12**
2017 Set-1	2	5	**12**
2017 Set-2	5	5	**15**
2018	6	5	**16**
2019	6	3	**12**
2020	3	5	**13**
2021 Set-1	5	6	**17**
2021 Set-2	5	4	**13**
2022	5	7	**19**
2023	6	5	**16**

1 Mathematical Logic

1.1 Let p and q be propositions. Using only the truth table decide whether $p \Leftrightarrow q$ does not imply $p \rightarrow \neg q$ is true or false. **[1994 : 2 M]**

1.2 If the proposition $\neg p \Rightarrow q$ is true, then the truth value of the proposition $\neg p \vee (p \Rightarrow q)$, where $\neg$ is negation, '$\vee$' is inclusive or and '$\Rightarrow$' is implication, is

(a) True
(b) Multiple valued
(c) False
(d) Cannot be determined

[1995 : 2 M]

1.3 Which one of the following is false? Read $\wedge$ as AND, $\vee$ as OR, ~ as NOT, $\rightarrow$ as one way implication and $\leftrightarrow$ as two way implication.

(a) $((x \rightarrow y) \wedge x) \rightarrow y$
(b) $((\sim x \rightarrow y) \wedge (\sim x \rightarrow \sim y)) \rightarrow x$
(c) $(x \rightarrow (x \vee y))$
(d) $((x \vee y) \leftrightarrow (\sim x \rightarrow \sim y))$ **[1996 : 2 M]**

1.4 Let a, b, c, d be propositions. Assume that the equivalence $a \leftrightarrow (b \vee \neg b)$ and $b \leftrightarrow c$ hold. Then the truth-value of the formula $(a \wedge b) \rightarrow (a \wedge c) \vee d$ is always

(a) True
(b) False
(c) Same as the truth-value of b
(d) Same as the truth-value of d **[2000 : 2 M]**

1.5 What is the converse of the following assertion?
I stay only if you go

(a) I stay if you go
(b) If I stay then you go
(c) If you do not go then I do not stay
(d) If I do not stay then you go **[2001 : 1 M]**

1.6 Consider two well-formed formulas in propositional logic:

$F_1 : P \Rightarrow \neg P$ $\quad$ $F_2 : (P \Rightarrow \neg P) \vee (\neg P \Rightarrow P)$

Which of the following statements is correct?

(a) F_1 is satisfiable, F_2 is valid
(b) F_1 is unsatisfiable, F_2 is satisfiable
(c) F_1 is unsatisfiable, F_2 is valid
(d) F_1 and F_2 are both satisfiable

[2001 : 1 M]

1.7 "If X then Y unless Z" is represented by which of the following formulas in propostional logic? ("$\neg$") is negation, "$\wedge$" is conjunction, and "$\rightarrow$" is implication)

(a) $(X \wedge \neg Z) \rightarrow Y$
(b) $(X \wedge Y) \rightarrow \neg Z$
(c) $X \rightarrow (Y \wedge \neg Z)$
(d) $(X \rightarrow Y) \wedge \neg Z$

[2002 : 1 M]

1.8 Which of the following is a valid first order formula? (Here α and β are first order formulae with x as their only free variable)

(a) $((\forall x)[\alpha] \Rightarrow (\forall x)[\beta]) \Rightarrow (\forall x)[\alpha \Rightarrow \beta]$
(b) $(\forall x)[\alpha] \Rightarrow (\exists x)[\alpha \wedge \beta]$
(c) $(\forall x)[\alpha \vee \beta] \Rightarrow (\exists x)[\alpha] \Rightarrow (\forall x)[\alpha]$
(d) $(\forall x)[\alpha \Rightarrow \beta] \Rightarrow ((\forall x)[\alpha] \Rightarrow (\forall x)[\beta])$

[2003 : 2 M]

1.9 Consider the following formula α and its two interpretations I_1 and I_2.

$\alpha : (\forall x)[P_x \Leftrightarrow (\forall y)[Q_{xy} \Leftrightarrow \neg Q_{yy}]] \Rightarrow (\forall x)[\neg P_x]$

I_1 **:** Domain : the set of natural numbers
$P_x \equiv$ 'x is a prime number'
$Q_{xy} \equiv$ 'y divides x'

I_2 **:** Same as I_1 except that $P_x =$ 'x is a composite number.'

Which of the following statements is true?

(a) I_1 satisfies α, I_2 does not
(b) I_2 satisfies α, I_1 does not
(c) Neither I_2 nor I_1 satisfies α
(d) Both I_1 and I_2 satisfy α **[2003 : 2 M]**

1.10 The following resolution rule is used in logic programming: Derive clause $(P \vee Q)$ from clauses $(P \vee R)$, $(Q \vee \neg R)$
Which of the following statemnets related to this rule is FALSE?

(a) $(P \vee R) \wedge (Q \vee \neg R) \Rightarrow (P \vee Q)$ is logically valid
(b) $(P \vee Q) \Rightarrow (P \vee R) \wedge (Q \vee \neg R)$ is logically valid
(c) $(P \vee Q)$ is satisfiable if and only if $(P \vee R) \wedge (Q \vee \neg R)$ is satisfiable
(d) $(P \vee Q) \Rightarrow$ FALSE if and only if both P and Q are unsatisfiable **[2003 : 2 M]**

1.11 Identify the correct translation into logical notation of the following assertion. Some boys in the class are taller than all the girls

Note: Taller (x, y) is true if x is taller than y.

(a) $(\exists x)$ (boy$(x) \to (\forall y)$ (girl$(y) \wedge$ taller (x, y)))
(b) $(\exists x)$ (boy$(x) \wedge (\forall y)$ (girl$(y) \wedge$ taller (x, y)))
(c) $(\exists x)$ (boy$(x) \to (\forall y)$ (girl$(y) \to$ taller (x, y)))
(d) $(\exists x)$ (boy$(x) \wedge (\forall y)$ (girl$(y) \to$ taller (x, y)))

[2004 : 1 M]

1.12 Let $a(x, y)$, $b(x, y,)$ and $c(x, y)$ be three statements with variables x and y chosen from some universe. Consider the following statement:

$$(\exists x)(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)]$$

Which one of the following is its equivalent?

(a) $(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \to c(x,y)]$
(b) $(\exists x)(\forall y)[(a(x,y) \vee b(x,y)) \wedge \neg\, c(x,y)]$
(c) $\neg(\forall x)(\exists y)[(a(x,y) \wedge b(x,y)) \to c(x,y)]$
(d) $\neg(\forall x)(\exists y)[(a(x,y) \vee b(x,y)) \to c(x,y)]$

[2004 : 1 M]

1.13 Let p, q, r and s be four primitive statements. Consider the following arguments:

P : $[(\neg p \vee q) \wedge (r \to s) \wedge (p \vee r)] \to (\neg s \to q)$
Q : $[(\neg p \wedge q) \wedge [q \to (p \to r)]] \to \neg r$
R : $[[(q \wedge r) \to p] \wedge (\neg q \vee p)] \to r$
S : $[p \wedge (p \to r) \wedge (q \vee \neg r)] \to q$

Which of the above arguments are valid?

(a) P and Q only
(b) P and R only
(c) P and S only
(d) P, Q, R and S

[2004 : 2 M]

1.14 The following propositional statement is

$$(\mathrm{P} \to (\mathrm{Q} \vee \mathrm{R})) \to ((\mathrm{P} \wedge \mathrm{Q}) \to \mathrm{R})$$

(a) Satisfiable but not valid
(b) Valid
(c) A contradititiction
(d) None of the above

[2004 : 2 M]

1.15 Let P, Q and R be three atomic propositional assertions. Let X denote $(\mathrm{P} \vee \mathrm{Q}) \to \mathrm{R}$ and Y denote $(\mathrm{P} \to \mathrm{R}) \vee (\mathrm{Q} \to \mathrm{R})$. Which one of the following is a tautology?

(a) $\mathrm{X} \equiv \mathrm{Y}$
(b) $\mathrm{X} \to \mathrm{Y}$
(c) $\mathrm{Y} \to \mathrm{X}$
(d) $\neg\,\mathrm{Y} \to \mathrm{X}$

[2005 : 2 M]

1.16 What is the first order predicate calculus statement equivalent to the following? Every teacher is liked by some student

(a) $\forall(x)$ [teacher $(x) \to \exists\,(y)$ [student $(y) \to$ likes (y, x)]]
(b) $\forall(x)$ [teacher $(x) \to \exists\,(y)$ [student $(y) \wedge$ likes (y, x)]]
(c) $\exists(y)\ \forall(x)$ [teacher $(x) \to$ [student $(y) \wedge$ likes (y, x)]]
(d) $\forall(x)$ [teacher $(x) \wedge \exists\,(y)$ [student $(y) \to$ likes (y, x)]]

[2005 : 2 M]

1.17 Let $P(x)$ and $Q(x)$ be arbitrary predicates. Which of the following statements is always TRUE?

(a) $(\forall x(P(x) \vee Q(x))) \Rightarrow ((\forall x P(x)) \vee (\forall x Q(x)))$
(b) $(\forall x(P(x) \Rightarrow Q(x))) \Rightarrow ((\forall x P(x)) \Rightarrow (\forall x Q(x)))$
(c) $(\forall x(P(x) \Rightarrow (\forall x Q(x)))) \Rightarrow (\forall x(P(x) \Rightarrow Q(x)))$
(d) $((\forall x(P(x)) \Leftrightarrow (\forall x Q(x))) \Rightarrow (\forall x(P(x) \Leftrightarrow Q(x)))$

[2005 : 2 M]

1.18 Consider the following first order logic formula in which R is a binary relation symbol.

$$\forall x\, \forall y\ (\ R(x,y) \Rightarrow R(y,x)\ )$$

The formula is

(a) Satisfiable and valid
(b) Satisfiable and so is its negation
(c) Unsatisfiable but its negation is valid
(d) Satisfiable but its negation is unsatisfiable

[2006 : 2 M]

1.19 Which one of the first order predicate calculus statements given below correctly expresses the following English statement?

Tigers and lions attack if they are hungry or threatened.

(a) $\forall x$ [(tiger $(x) \wedge$ lion (x)) $\to$ {(hungry $(x) \vee$ threatened (x)) $\to$ attacks (x)}]
(b) $\forall x$ [(tiger $(x) \vee$ lion (x)) $\to$ {(hungry $(x) \vee$ threatened (x)) $\wedge$ attacks (x)}]
(c) $\forall x$ [(tiger $(x) \vee$ lion (x)) $\to$ {(attacks $(x) \to$ (hungry $(x) \vee$ Threatened (x))}]
(d) $\forall x$ [(tiger $(x) \vee$ lion (x)) $\to$ {(hungry $(x) \vee$ threatened (x)) $\to$ attacks (x)}]

[2006 : 2 M]

1.20 Consider the following propositional statements:

$\boldsymbol{P_1}$ **:** $((A \wedge B) \to C)) \equiv ((A \to C) \wedge (B \to C))$
$\boldsymbol{P_2}$ **:** $((A \vee B) \to C)) \equiv ((A \to C) \vee (B \to C))$

Which one of the following is true?

(a) P_1 is a tautology, but not P_2
(b) P_2 is a tautology, but not P_1
(c) P_1 and P_2 are both tautologies
(d) Both P_1 and P_2 are not tautologies

[2006 : 2 M]

1.21 A logical binary relation $\odot$, is defined as follows:

A	B	A $\odot$ B
True	True	True
True	False	True
False	True	False
False	False	True

Let ~ be the unary negation (NOT) operator, with higher precedence, than $\odot$. Which one of the following is equivalent to $\mathrm{A} \wedge \mathrm{B}$?

(a) $(\sim A \odot B)$ (b) $\sim (A \odot \sim B)$
(c) $\sim (\sim A \odot \sim B)$ (d) $\sim (\sim A \odot B)$

[2006 : 2 M]

1.22 Let Graph (x) be a predicate which denotes that x is a graph. Let Connected (x) be a predicate which denotes that x is connected. Which of the following first order logic sentences DOES NOT represent the statement; "Not every graph is connected"?

(a) $\neg \forall x$ (Graph(x) $\Rightarrow$ Connected (x))
(b) $\exists x$ (Graph(x) $\wedge \neg$ Connected (x))
(c) $\neg \forall x$ ($\neg$ Graph(x) $\vee$ Connected (x))
(d) $\forall x$ (Graph(x) $\Rightarrow \neg$ Connected (x))

[2007 : 2 M]

1.23 Which of the following is TRUE about formulae in Conjunctive Normal Form?

(a) For any formula, there is a truth assignment for which at least half the clauses evaluate to true.
(b) For any formula, there is a truth assignment for a which all the clauses evaluate to true.
(c) There is a formula such that for each truth assignment at most one-fourth of the clauses evaluate to true.
(d) None of the above **[2007 : 2 M]**

1.24 Which one of these first-order logic formulae is valid?

(a) $\forall x(P(x) \Rightarrow Q(x)) \Rightarrow ((\forall x P(x)) \Rightarrow (\forall x Q(x)))$
(b) $\exists x(P(x) \vee Q(x)) \Rightarrow ((\exists x P(x)) \Rightarrow (\exists x Q(x)))$
(c) $\exists x(P(x) \wedge Q(x)) \Leftrightarrow ((\exists x P(x)) \wedge (\exists x Q(x)))$
(d) $\forall x \exists y \ P(x, y) \Rightarrow \exists y \forall x P(x, y)$

[2007 : 2 M]

1.25 Let *fsa* and *pda* be two predicates such that *fsa*(x) means x is a finite state automaton, and *pda*(y) means, that y is a pushdown automaton. Let equivalent be another predicate such that equivalent (a, b) means a and b are equivalent. Which of the following first order logic statement represents the following:

Each finite state automaton has an equivalent pushdown automaton.

(a) $\forall x\ (fsa(x) \Rightarrow \exists y\ (pda(y) \wedge$ equivalent $(x, y)))$
(b) $\sim\forall y(\exists x\ fsa(x) \Rightarrow pda(y) \wedge$ equivalent $(x, y))$
(c) $\forall x\ \exists y\ (fsa(x) \wedge pda(y) \wedge$ equivalent $(x, y))$
(d) $\forall x\ \exists y\ (fsa(y) \wedge pda(x) \wedge$ equivalent $(x, y))$

[2008 : 1 M]

1.26 Which of the following first order formulae is logically valid? Here $\alpha(x)$ is a first order formula with x as a free variable, and β is a first order formula with no free variable.

(a) $[\beta \rightarrow (\exists x, \alpha(x))] \rightarrow [\forall x, \beta \rightarrow \alpha(x)]$
(b) $[\exists x, \beta \rightarrow \alpha(x)] \rightarrow [\beta \rightarrow (\forall x, \alpha(x))]$
(c) $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$
(d) $[(\forall x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

[2008 : 2 M]

1.27 Which of the following is the negation of $[(\forall x, \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u, \exists v, \gamma))]$

(a) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \gamma))]$
(b) $[\exists x, \alpha \rightarrow (\forall y, \beta \rightarrow (\exists u, \forall v, \neg\gamma))]$
(c) $[\forall x, \neg\alpha \rightarrow (\exists y, \neg\beta \rightarrow (\forall u, \exists v, \neg\gamma))]$
(d) $[\forall x, \alpha \wedge (\exists y, \beta \wedge (\exists u, \forall v, \neg\gamma))]$

[2008 : 2 M]

1.28 P and Q are two propositions. which of the following logical expressions are equivalent?

1. $P \vee \sim Q$
2. $\sim (\sim P \wedge Q)$
3. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$
4. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(a) Only 1 and 2 (b) Only 1, 2 and 3
(c) Only 1, 2 and 4 (d) All of these

[2008 : 2 M]

1.29 Which one of the following is the most appropriate logical formula to represent the statement:
"Gold and silver ornaments are precious"
The following notations are used:
$G(x)$: x is a gold ornament
$S(x)$: x is a silver ornament
$P(x)$: x is precious

(a) $\forall x(P(x) \rightarrow (G(x) \wedge S(x)))$
(b) $\forall x(G(x) \wedge (S(x) \rightarrow P(x)))$
(c) $\exists x((G(x) \wedge S(x)) \rightarrow P(x))$
(d) $\forall x((G(x) \vee S(x)) \rightarrow P(x))$ **[2009 : 2 M]**

1.30 The binary operation $\square$ is defined as follows:

P	Q	P$\square$Q
T	T	T
T	F	T
F	T	F
F	F	T

Which one of the following is equivalent to $P \vee Q$?

(a) $\neg Q \square \neg P$ (b) $P \square \neg Q$
(c) $\neg P \square Q$ (d) $\neg P \square \neg Q$

[2009 : 2 M]

1.31 Consider the following well-formed formulae:

I. $\neg\forall x(P(x))$ **II.** $\neg\exists x(P(x))$

III. $\neg\exists x(\neg P(x))$ **IV.** $\exists x(\neg P(x))$

Which of the above are equivalent?

(a) I and III (b) I and IV

(c) II and III (d) II and IV

[2009 : 2 M]

1.32 Suppose the predicate $F(x, y, t)$ is used to represent the statement that person x can fool person y at time t. Which one of the statements below expresses best the meaning of the formula $\forall x\exists y\exists t\ (\neg F(x, y, t))$?

(a) Everyone can fool some person at some time

(b) No one can fool everyone all the time

(c) Everyone cannot fool some person all the time

(d) No one can fool some person at some time

[2010 : 2 M]

1.33 Which one of the following options is CORRECT given three positive integers x, y and z, and a predicate

$$P(x) = \neg(x = 1) \wedge \forall y(\exists z(x = y * z) \Rightarrow (y = x) \vee (y = 1)$$

(a) $P(x)$ being true means that x is a prime number

(b) $P(x)$ being true means that x is a number other than 1

(c) $P(x)$ is always true irrespective of the value of x

(d) $P(x)$ being true means that x has exactly two factors other than 1 and x **[2011 : 2 M]**

1.34 Consider the following logical inferences.

I_1 : If it rains then the cricket match will not be played.
The cricket match was played.
Inference: There was no rain.

I_2 : If it rains then the cricket match will not be played.
It did not rain.
Inference: The cricket match was played.

Which of the following is TRUE?

(a) Both I_1 and I_2 are correct inferences

(b) I_1 is correct but I_2 is not a correct inference

(c) I_1 is not correct but I_2 is a correct inference

(d) Both I_1 and I_2 are not correct inferences

[2012 : 1 M]

1.35 What is the correct translation of the following statement into mathematical logic?

"Some real numbers are rational"

(a) $\exists x$ (real $(x) \vee$ rational (x))

(b) $\forall x$ (real $(x) \rightarrow$ rational (x))

(c) $\exists x$ (real $(x) \wedge$ rational (x))

(d) $\exists x$ (rational $(x) \rightarrow$ real (x)) **[2012 : 1 M]**

1.36 What is the logical translation of the following statements?

"None of my friends are perfect"

(a) $\exists x\ (F(x) \wedge \neg P(x))$ (b) $\exists x\ (\neg F(x) \wedge P(x))$

(c) $\exists x\ (\neg F(x) \wedge \neg P(x))$ (d) $\neg\exists x\ (F(x) \wedge P(x))$

[2013 : 2 M]

1.37 Which one of the following is NOT logically equivalent to $\neg\exists x(\forall y(\alpha) \wedge \forall z(\beta))$

(a) $\forall x(\exists z(\neg\beta) \rightarrow \forall y(\alpha))$

(b) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg\alpha))$

(c) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg\beta))$

(d) $\forall x(\exists y(\neg\alpha) \vee \exists z(\neg\beta))$ **[2013 : 2 M]**

1.38 Consider the statement:

"Not all that glitters is gold"

Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement?

(a) $\forall x :$ glitters$(x) \Rightarrow \neg$gold(x)

(b) $\forall x :$ gold$(x) \Rightarrow$ glitters(x)

(c) $\exists x :$ gold$(x) \wedge \neg$glitters(x)

(d) $\exists x :$ glitters$(x) \wedge \neg$gold(x)

[2014 (Set-1) : 1 M]

1.39 Which one of the following propositional logic formulas is TRUE when exactly two of p, q, and r are **TRUE**?

(a) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(b) $(\sim(p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(c) $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(d) $(\sim(p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

[2014 (Set-1) : 2 M]

1.40 Which one of the following Boolean expressions is NOT a tautology?

(a) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$

(b) $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$

(c) $(a \wedge b \wedge c) \rightarrow (c \vee a)$

(d) $a \rightarrow (b \rightarrow a)$ **[2014 (Set-2) : 2 M]**

1.41 Consider the following statements:

P : Good mobile phones are not cheap

Q : Cheap mobile phones are not good

L : P implies Q

M : Q implies P

N : P is equivalent to Q

Which one of the following about L, M, and N is **CORRECT**?

(a) Only L is TRUE (b) Only M is TRUE
(c) Only N is TRUE (d) L, M and N are TRUE.
[2014 (Set-3) : 1 M]

1.42 The CORRECT formula for the sentence, "not all rainy days are cold" is
(a) $\forall d$(Rainy $(d)\wedge$~Cold (d))
(b) $\forall d$(~Rainy $(d) \to$ Cold (d))
(c) $\exists d$(~Rainy $(d) \to$ Cold (d))
(d) $\exists d$(Rainy $(d)\wedge$ ~ Cold (d))
[2014 (Set-3) : 2 M]

1.43 Which one of the following is Not equivalent to $p \leftrightarrow q$?
(a) $(\neg p \vee q) \wedge (p \vee \neg q)$
(b) $(\neg p \vee q) \wedge (q \to p)$
(c) $(\neg p \wedge q) \vee (p \wedge \neg q)$
(d) $(\neg p \wedge \neg q) \vee (p \wedge q)$ **[2015 (Set-1) : 1 M]**

1.44 The binary operator $\neq$ is defined by the following truth table.

p	q	$p \neq q$
0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the binary operator $\neq$?
(a) Both commutative and associative
(b) Commutative but not associative
(c) Not commutative but associative
(d) Neither commutative nor associative
[2015 (Set-1) : 2 M]

1.45 Consider the following two statements:
S_1: If a candidate is known to be corrupt, then he will not be elected.
S_2: If a candidate is kind, he will be elected.
Which one of the following statements follows from S_1 and S_2 as per sound inference rules of logic?
(a) If a person is known to be corrupt, he is kind
(b) If a person is not known to be corrupt, he is not kind
(c) If a person is kind, he is not known to be corrupt
(d) If a person is not kind, he is not known to be corrupt **[2015 (Set-2) : 1 M]**

1.46 Which one of the following well formed formulae is a tautology?
(a) $\forall x\ \exists y\ R(x, y) \leftrightarrow \exists y\ \forall x\ R(x, y)$
(b) $(\forall x\ [\exists y\ R(x, y) \to S(x, y)]) \to \forall x\ \exists y\ S(x, y)$
(c) $[\forall x\ \exists y\ (P(x, y) \to R(x, y))] \leftrightarrow [\forall x\ \exists y\ (\neg P(x, y) \vee R(x, y))]$
(d) $\forall x\ \forall y\ P(x, y) \to \forall x\ \forall y\ P(y, x)$
[2015 (Set-2) : 2 M]

1.47 In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking, the person replies the following:
"The result of the toss is head if and only if I am telling the truth."
Which of the following options is correct?
(a) The result is head
(b) The result is tail
(c) If the person is of Type 2, then the result is tail
(d) If the person is of Type 1, then the result is tail **[2015 (Set-3) : 1 M]**

1.48 Let p, q, r, s represent the following propositions.
p : $x \in \{8, 9, 10, 11, 12\}$
q : x is a composite number
r : x is a perfect square
s : x is a prime number
The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is _____. **[2016 (Set-1) : 1 M]**

1.49 Consider the following expressions:
(i) False (ii) Q
(iii) True (iv) $P \vee Q$
(v) $\neg Q \vee P$
The number of expressions given above that are logically implied by $P\wedge(P \Rightarrow Q)$ is ______.
[2016 (Set-2) : 1 M]

1.50 Which one of the following well-formed formulae in predicate calculus is **NOT** valid?
(a) $(\forall x\, p(x) \Rightarrow \forall x\, q(x)) \Rightarrow (\exists x \neg p(x) \vee \forall x\, q(x))$
(b) $(\exists x\, p(x) \vee \exists x\, q(x)) \Rightarrow \exists x(p(x) \vee q(x))$
(c) $\exists x\,(p(x) \wedge q(x)) \Rightarrow (\exists x\, p(x) \wedge \exists x\, q(x))$
(d) $\forall x\,(p(x) \vee q(x)) \Rightarrow (\forall x\, p(x) \vee \forall x\, q(x))$
[2016 (Set-2) : 2 M]

1.51 Consider the first-order logic sentence $F : \forall x(\exists y R(x, y))$. Assuming non-empty logical domains, which of the sentences below are implied by F?
I. $\exists y(\exists x\ R(x, y))$ **II.** $\exists y(\forall x\ R(x, y))$
III. $\forall y(\exists x\ R(x, y))$ **IV.** $\neg\exists x(\forall y \neg R(x, y))$

(a) IV only (b) I and IV only
(c) II only (d) II and III only

[2017 (Set-1) : 1 M]

1.52 The statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statements below?

I. $p \Rightarrow q$ **II.** $q \Rightarrow p$
III. $(\neg q) \vee p$ **IV.** $(\neg p) \vee q$

(a) I only (b) I and IV only
(c) II only (d) II and III only

[2017 (Set-1) : 1 M]

1.53 Let p, q and r be propositions and the expression $(p \to q) \to r$ be a contradiction. Then, the expression $(r \to p) \to q$ is
(a) A tautology
(b) A contradiction
(c) Always TRUE when p is FALSE
(d) Always TRUE when q is TRUE

[2017 (Set-1) : 2 M]

1.54 Let p, q, r denote the statements "*It is raining*", "*It is cold*", and "*It is pleasant*", respectively. Then the statement "*It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold*" is represented by
(a) $(\neg p \wedge r) \wedge (\neg r \to (p \wedge q))$
(b) $(\neg p \wedge r) \wedge ((p \wedge q) \to \neg r)$
(c) $(\neg p \wedge r) \vee ((p \wedge q) \to \neg r)$
(d) $(\neg p \wedge r) \vee (r \to (p \wedge q))$

[2017 (Set-2) : 1 M]

1.55 Consider the first order predicate formula φ:
$\forall x[(\forall z\; z | x \Rightarrow ((z = x) \vee (z = 1))) \Rightarrow \exists w\; (w > x) \wedge (\forall z\; z | w \Rightarrow ((w = z) \vee (z = 1)))]$
Here '$a | b$' denotes that 'a divides b', where a and b are integers. Consider the following sets:
S_1 : {1, 2, 3, ..., 100}
S_2 : Set of all positive integers
S_3 : Set of all integers
Which of the above sets satisfy φ?
(a) S_1 and S_3 (b) S_2 and S_3
(c) S_1, S_2 and S_3 (d) S_1 and S_2

[2019 : 2 M]

1.56 Which one of the following predicate formulae is NOT logically valid?
Note that W is a predicate formula without any free occurrence of x.
(a) $\exists x(p(x) \wedge W) \equiv \exists x\; p(x) \wedge W$
(b) $\forall x(p(x) \to W) \equiv \forall x\; p(x) \to W$
(c) $\exists x(p(x) \to W) \equiv \forall x\; p(x) \to W$
(d) $\forall x(p(x) \vee W) \equiv \forall x\; p(x) \vee W$ **[2020 : 2 M]**

1.57 Let p and q be two propositions. Consider the following two formulae in prepositional logic.
S_1 : $(\neg p \wedge (p \vee q)) \to q$
S_2 : $q \to (\neg p \wedge (p \vee q))$
Which one of the following choices is correct?
(a) Neither S_1 nor S_2 is a tautology.
(b) Both S_1 and S_2 are tautologies.
(c) S_1 is a tautology but S_2 is not a tautology.
(d) S_1 is not a tautology but S_2 is a tautology.

[2021 (Set-1) : 1 M]

1.58 Choose the correct choice(s) regarding the following propositional logic assertion S:
$S : ((P \wedge Q) \to R) \to ((P \wedge Q) \to (Q \to R))$
(a) S is a tautology.
(b) S is neither a tautology nor a contradiction.
(c) The antecedent of S is logically equivalent to the consequent of S.
(d) S is a contradiction.

[2021 (Set-2) : 1 M]

1.59 Geetha has a conjecture about integers, which is of the form $\forall x\big(P(x) \Rightarrow \exists y\, Q(x, y)\big)$, where P is a statement about integers, and Q is a statement about pairs of integers.
Which of the following (one or more) option(s) would imply Geetha's conjecture?
(a) $\exists x\big(P(x) \wedge \forall y\, Q(x, y)\big)$
(b) $\forall x \forall y\, Q(x, y)$
(c) $\exists y\, \forall x\big(P(x) \Rightarrow Q(x, y)\big)$
(d) $\exists x\big(P(x) \wedge \exists y\, Q(x, y)\big)$ **[2023 : 1 M]**

■■■■

Answers Mathematical Logic

1.1	Sol.	**1.2**	(d)	**1.3**	(d)	**1.4**	(a)	**1.5**	(a)	**1.6**	(a)	**1.7**	(a)	**1.8**	(d)	**1.9**	(d)
1.10	(b)	**1.11**	(d)	**1.12**	(c)	**1.13**	(c)	**1.14**	(a)	**1.15**	(b)	**1.16**	(b)	**1.17**	(b)	**1.18**	(b)
1.19	(d)	**1.20**	(d)	**1.21**	(d)	**1.22**	(d)	**1.23**	(a)	**1.24**	(a)	**1.25**	(a)	**1.26**	(c)	**1.27**	(d)
1.28	(b)	**1.29**	(d)	**1.30**	(b)	**1.31**	(b)	**1.32**	(b)	**1.33**	(a)	**1.34**	(b)	**1.35**	(c)	**1.36**	(d)
1.37	(a)	**1.38**	(d)	**1.39**	(b)	**1.40**	(b)	**1.41**	(d)	**1.42**	(d)	**1.43**	(c)	**1.44**	(a)	**1.45**	(c)
1.46	(c)	**1.47**	(a)	**1.48**	(11)	**1.49**	(4)	**1.50**	(d)	**1.51**	(b)	**1.52**	(d)	**1.53**	(d)	**1.54**	(a)
1.55	(b)	**1.56**	(b)	**1.57**	(c)	**1.58**	(a, c)	**1.59**	(b, c)								

Explanations Mathematical Logic

1.1 Sol.

TRUE

p	q	$p \leftrightarrow q$	$p \to \sim q$	$(p \leftrightarrow q) \to (p \to \sim q)$
T	T	T	F	F
T	F	F	T	T
F	T	F	T	T
F	F	T	T	T

From the truth table, $(p \leftrightarrow q) \to (p \to \sim q)$ is not tautology, hence it is true that $p \leftrightarrow q$ doesn't imply $p \to \neg q$.

1.2 (d)

p	q	$\neg p \to q$
0	0	0
0	1	1
1	0	1
1	1	1

Now since $\neg p \to q$ is given true, we reduce the truth table as follows:

p	q	$\neg p \to q$
0	1	1
1	0	1
1	1	1

In the reduced truth table we need to find the truth value of $\neg p \vee (p \to q) \equiv p' + (p \to q)$

$\equiv p' + p' + q \equiv p' + q$

The truth value of p′ + q in the reduced truth table is given below:

p	q	$p' + q$
0	1	1
1	0	0
1	1	1

Since in the reduced truth table also, the given expression is sometimes true and sometimes false, therefore the truth value of proposition $\neg p \vee (p \to q)$ can not be determined.

1.3 (d)

(a) $((x \to y) \wedge x) \to y$

$= \sim((\sim x \vee y) \wedge x) \vee y$

$= \sim((\sim x \wedge x) \vee (y \wedge x)) \vee y$

$= \sim(F \vee (y \wedge x) \vee y$

$= \sim(y \wedge x) \vee y = \sim y \vee \sim x \vee y$

$= (\sim y \vee y) \vee \sim x$

$= T \vee \sim x = T$

(b) $((\sim x \to y) \wedge (\sim x \to \sim y)) \to x$

$= \sim(\sim(\sim x) \vee y) \wedge (\sim(\sim x) \vee \sim y)) \vee x$

$= \sim((x \vee y) \wedge (x \vee \sim y)) \vee x$

$= \sim(x \vee (y \wedge \sim y)) \vee x = \sim(x \vee F) \vee x$

$= \sim x \vee x = T$

(c) $(x \to (x \vee y))$

$= (\sim x \vee (x \vee y)) = ((\sim x \vee x) \vee y)$

$= (T \vee y) = T$

(d) $((x \vee y) \leftrightarrow (\sim x \to \sim y))$

$= (x \vee y) \leftrightarrow (\sim(\sim x) \vee \sim y)$

$= (x \vee y) \leftrightarrow (x \vee \sim y)$

$= ((x \vee y) \wedge (x \vee \sim y)) \vee (\sim(x \vee y) \wedge \sim(x \vee \sim y))$

$= ((x \vee (y \wedge \sim y)) \vee ((\sim x \wedge \sim y) \wedge (\sim x \wedge y))$

$= (x \vee F) \vee ((\sim x \wedge \sim y) \wedge (\sim x \wedge y))$

$= x \vee (\sim x \wedge (y \wedge \sim y)) = x \vee (\sim x \wedge F)$

$= x \vee F = x$

1.4 (a)

- $a \leftrightarrow (b \vee \neg b)$
  $a \leftrightarrow$ True
  So a is true, i.e. $a = 1$
- $b \leftrightarrow c$ holds. So $b = c$
  Now the given expression is
  $(a \wedge b) \to ((a \wedge c) \vee d) \equiv (a \cdot b) \to ((a \cdot c) + d)$
  Putting a = 1 in above expression we get
  $1 \cdot b \to ((1 \cdot c) + d)$
  $\equiv b \to c + d \equiv b' + c + d$

Now putting $b = c$ in above expression we get

$\equiv c' + c + d \equiv 1 + d \equiv 1$

So the expression is always true.

1.5 (a)

Let, p : I stay, q : you go

I stay only if you go $p \to q$

Converse of $p \to q$ is $q \to p$

Now convert the answers one-by-one into boolean form. Only option (a) i.e. "*I stay if you go*" converts to $q \to p$.

1.6 (a)

$F_1 : P \to {\sim}P \equiv p \to p' \equiv p' + p' \equiv p'$

So F_1 is contingency. Hence, F_1 is satisfiable but not valid.

$F_2 : (P \to {\sim}P) \vee ({\sim}P \to P)$

$\equiv (p \to p') + (p' \to p) \equiv (p' + p') + (p + p)$

$\equiv p' + p \equiv 1$

So F_2 is tautology and therefore valid.

1.7 (a)

If X then Y unless Z is represented by

$X \to Y$ unless $Z \equiv (X \to Y) + Z \equiv X' + Y + Z$

Now convert the answers one-by-one into boolean form only choice (a) converts to $X' + Y + Z$ as can be seen below:

$$(X \wedge \neg Z) \to Y \equiv XZ' \to Y$$
$$\equiv (XZ')' + Y \equiv X' + Y + Z$$

1.8 (d)

This is valid LHS is saying that if α is holding for any x, then β also holds for that x. RHS is saying if x is holding for all x, then β also holds for all x.

Clearly LHS $\Rightarrow$ RHS (but RHS does not imply LHS)

$$\underbrace{\underbrace{[\underbrace{(\underbrace{\alpha_1}_{T} \to \underbrace{\beta_1}_{F})}_{F} \wedge (\alpha_2 \to \beta_2)]}_{F} \to \underbrace{[\underbrace{(\alpha_1 \wedge \alpha_2)}_{\underbrace{T \wedge T}_{T}} \to \underbrace{(\beta_1 \wedge \beta_2)}_{\underbrace{F}_{F}}]}_{F}}_{T}$$

Let: $\alpha_1, \alpha_2 : T$

$\beta_1 : F$

$\Downarrow$

Valid

1.9 (d)

$Q_{yy} \equiv$ "y divides y" is always true

$\therefore\ Q_{xy} \Leftrightarrow \neg\, Q_{yy}$ is same as $Q_{xy} \Leftrightarrow$ False

Now α becomes

$(\forall x)[P(x) \Leftrightarrow (\forall y)(Q_{xy} \Leftrightarrow \text{false})]$

$\Rightarrow (\forall x)\ [\neg\, P(x)]$

Now consider $I_1 : P(x) \equiv$ "x is a prime number".

α becomes ($\forall x$ x is a prime number if and only if $\forall y$ (y does not divide x)) $\Rightarrow (\forall x)$ x is not prime.

which means that x is a prime number if and only if no number divides x implies that no number is prime.

Since x always divides x, the above sentence is true.

Now consider $I_2 : P(x) \equiv$ "x is a composite number".

Now α becomes

($\forall x$ x is a composite number if and only if $\forall y$ (y does not divide x)) $\Rightarrow (\forall x)$ x is not composite.

Which means that x is a composite number if and only if no number divides x implies that no number is composite.

Since x always divides x, the above sentence is true.

$\therefore$ Both I_1 and I_2 satisfy α.

1.10 (b)

Derive clause $P \vee Q$ from clauses $P \vee R$, $Q \vee \neg R$ means that $(P \vee R) \wedge (Q \vee \neg R) \Rightarrow P \vee Q$

$\therefore$ (a) is true

Since, $x \Rightarrow y$ does not imply that $y \Rightarrow x$

$\therefore\ P \vee Q \Rightarrow (P \vee R) \wedge (Q \vee \neg R)$

$\therefore$ may or may not be true.

Hence (b) is false.

1.11 (d)

The statement is "some boys in the class are taller than all the girls".

So the notation for the given statement is

$(\exists x)$ (boy(x) $\wedge$ $(\forall y)$ (girl(y) $\to$ taller (x, y)))

1.12 (c)

Choice (c) is

$$\neg(\forall x)(\exists y)[(a(x,y) \wedge b(x,y)) \to c(x,y)]$$
$$\equiv \neg(\forall x)(\exists y)[a \wedge b \to c]$$
$$\equiv \neg(\forall x)(\exists y)[(ab)' + c]$$
$$\equiv \exists x\, \forall y[(ab)' + c]'$$
$$\equiv \exists x\, \forall y[abc'] \equiv \exists x\, \forall y[a \wedge b \wedge \neg c]$$

which is same as the given expression.

$$(\exists x)(\forall y)[(a(x,y) \wedge b(x,y)) \wedge \neg c(x,y)]$$

1.13 (c)

P: $[(\neg p \vee q) \wedge (r \to s) \wedge (p \vee r)] \to (\neg s \to q)$

$\equiv [(p \to q) \wedge (r \to s) \wedge (p \vee r)] \to (q \vee s)$

which is a rule of inference called constructive dilemma and therefore valid.

S: $[p \wedge (p \to r) \wedge (q \vee \neg r)] \to q$

$\equiv p(p' + r)(q + r') \to q$

$\equiv pr(q + r') \to q \equiv prq \to q$

$\equiv (prq)' + q \equiv p' + r' + q' + q$

$\equiv p' + r' + 1 \equiv 1$

Therefore S is valid.

Q and R can be similarly simplified in boolean algebra to show that they are both not equivalent to 1.
So only P and S are valid.

1.14 (a)

$(P \to (Q \lor R)) \to ((P \land Q) \to R)$
$\equiv (P \to Q + R) \to (PQ \to R)$
$\equiv [P' + Q + R] \to [(PQ)' + R]$
$\equiv [P' + Q + R] \to [P' + Q' + R]$
$\equiv (P' + Q + R)' + P' + Q' + R$
$\equiv P\,Q'\,R' + P' + Q' + R$
$\equiv Q' + Q'\,P\,R' + P' + R$
$\equiv Q' + P' + R$ (by absorption law)
Which is a contingency (i.e. satisfiable but not valid).

1.15 (b)

X: $(P \lor Q) \to R$
Y: $(P \to R) \lor (Q \to R)$
X: $P + Q \to R \equiv (P + Q)' + R \equiv P'\,Q' + R$
Y: $(P' + R) + (Q' + R) \equiv P' + Q' + R$
Clearly $X \neq Y$
Consider $X \to Y$
$\equiv (P'Q' + R) \to (P' + Q' + R)$
$\equiv (P'Q' + R)' + P' + Q' + R$
$\equiv (P'Q')' \cdot R' + P' + Q' + R$
$\equiv (P + Q) \cdot R' + P' + Q' + R$
$\equiv PR' + QR' + P' + Q' + R$
$\equiv (PR' + R) + (QR' + Q') + P'$
$\equiv (P + R)\,(R' + R) + (Q + Q') \times (R' + Q') + P'$
$\equiv (P + R) + (R' + Q') + P'$
$\equiv P + P' + R + R' + Q' \equiv 1 + 1 + Q' \equiv 1$
$\therefore X \to Y$ is a tautology.

1.16 (b)

Every teacher is liked by some student: then the logical expression is $\forall(x)$ [teacher$(x) \to \exists(y)$ [student $(y) \land$ likes (y, x)]]
Where likes (y, x) means y likes x, such that y represent the student and x represents the teacher.

1.17 (b)

Consider choice (b)
$(\forall x(P(x) \Rightarrow Q(x))) \Rightarrow ((\forall x P(x)) \Rightarrow (\forall x Q(x)))$
Let the LHS of this implication be true
This means that

$$P_1 \to Q_1$$
$$P_2 \to Q_2$$
$$\vdots$$
$$P_n \to Q_n$$

Now we need to check if the RHS is also true.
The RHS is $((\forall x P(x)) \Rightarrow (\forall x Q(x)))$
To check this let us take the LHS of this as true i.e. take $\forall x\, P(x)$ to be true. This means that $(P_1, P_2, \ldots P_n)$ is taken to be true. Now P_1 along with $P_1 \to Q_1$ will imply that Q_1 is true. Similarly P_2 along with $P_2 \to Q_2$ will imply that Q_2 is true. And so on...
Therefore $(Q_1, Q_2, \ldots Q_n)$ all true.
i.e. $\forall x\, Q(x)$ is true. Therefore the statement (b) is a valid predicate statement.

1.18 (b)

Since a relation may or may not be symmetric, the given predicate is satisfiable but not valid. So (a) is clearly false.
Whenever a predicate is satisfiable its negation also is satisfiable. So option (b) is the correct answer.

1.19 (d)

The given statement should be read as
"If an animal is a tiger or a lion, then (if the animal is hungry or threatened, then it will attack). Therefore the correct translation is
$\forall x$ [(tiger $(x) \lor$ lion $(x)) \to${(hungry $(x) \lor$ threatened $(x)) \to$ attacks (x)}]
which is choice (d).

1.20 (d)

$P_1 : ((A \land B) \to C)) \equiv ((A \to C) \land (B \to C))$
LHS :
$(A \land B) \to C$
$\equiv AB \to C \equiv (A\,B)' + C$
$\equiv A' + B' + C$
RHS :
$(A \to C) \land (B \to C)$
$\equiv (A' + C)\,(B' + C) \equiv A'B' + C$
Clearly, LHS $\neq$ RHS
P_1 is not a tautology
$P_2 : ((A \lor B \to C)) \equiv ((A \to C) \lor (B \to C))$
LHS $\equiv (A + B \to C)$
$\equiv (A + B)' + C$
$\equiv A'B' + C$
RHS $\equiv (A \to C) \lor (B \to C)$
$\equiv (A' + C) + (B' + C)$
$\equiv A' + B' + C$
Clearly, LHS $\neq$ RHS $\Rightarrow P_2$ is also not a tautology.
Therefore, both P_1 and P_2 are not tautologies.
Correct choice is (d).

1.21 (d)

By using min terms we can define

$A \odot B = AB + AB' + A'B' = A + A'B'$
$= (A + A') \cdot (A + B') = A + B'$

(a) $\sim A \odot B = A' \odot B = A' + B'$

(b) $\sim (A \odot \sim B) = (A \odot B')'$
$= (A + (B')')'$
$= (A + B)' = A'B'$

(c) $\sim (\sim A \odot \sim B) = (A' \odot B')' = (A' + (B')')'$
$= (A' + B)' = AB'$

(d) $\sim (\sim A \odot B) = (A' \odot B)' = (A' + B')'$
$= A \cdot B = A \wedge B$

$\therefore$ Only, choice (d) $\equiv A \wedge B$

Note: This problem can also be done by constructing truth table for each choice and comparing with truth table for $A \wedge B$.

1.22 (d)

The statement "Not every graph is connected" is same as "There exists some graph which is not connected" which is same as

$\exists x$ {graph $(x) \wedge \neg$ connected (x)}

Which is choice (b)

By boolean algebra we can see that option (a) and (c) are same as (b). Only option (d) is not the same as (b).

Infact option (d) means that "all graphs are not connected".

Alternate solution:

We can translate the given statement "NOT (every graph is connected)" as $\neg(\forall x$ graph (x) $\rightarrow$ connected $(x))$

$\equiv \exists x \neg$(graph $(x) \rightarrow$ connected (x))

$\equiv \exists x \neg(\neg$graph $(x) \vee$ connected (x))

$\equiv \exists x$ (graph $(x) \wedge \neg$ connected (x))

By boolean algebra we can see that option (a) and (c) are same as (b). Only option (d) is not the same as (b).

Infact option (d) means that "all graphs are not connected".

1.23 (a)

In conjunction normal form, for any particular assignment of truth values, all except one clause, will always evaluate to true. So, the proportion of clauses which evaluate to true to the total number of clauses is equal to $\frac{2^n - 1}{2^n}$.

Now putting $n = 1, 2, \ldots,$ we get $\frac{1}{2}, \frac{3}{4}, \frac{7}{8} \ldots$

All of these proportions are $\geq \frac{1}{2}$ and so choice (a) atleast half of the clauses evaluate to true, is the correct answer.

1.24 (a)

Option (a) is a standard one way distributive property of predicates.

1.25 (a)

"For x which is an *fsa*, there exists a y which is a *pda* and which is equivalent to x."

$\forall x\ (fsa(x) \Rightarrow \exists y\ (pda(y) \wedge \text{equivalent } (x, y)))$ is the logical representation.

1.26 (c)

Option (c) is $[(\exists x, \alpha(x)) \rightarrow \beta] \rightarrow [\forall x, \alpha(x) \rightarrow \beta]$

Let us check the validity of this predicate.

Let the LHS of this predicate be true.

This means that some $\alpha \rightarrow \beta$.

Let $\alpha_5 \rightarrow \beta$

Now we will check if the RHS is true. The RHS is $[\forall x, \alpha(x) \rightarrow \beta]$ to check this implication let us take $\forall x, \alpha(x)$ to be true.

This means that all the α are true. It means that α_5 is also true.

But $\alpha_5 \rightarrow \beta$. Therefore β is true.

So the RHS $[\forall x, \alpha(x) \rightarrow \beta]$ is true.

Whenever the LHS $[(\exists x, \alpha(x)) \rightarrow \beta]$ is true. So option (c) is valid.

1.27 (d)

The given predicate is

$[\forall x,\ \alpha \rightarrow (\exists y, \beta \rightarrow (\forall u,\ \exists v,\ \gamma))]$

The negation is of this predicate is

$\neg[\forall x,\ \alpha \rightarrow (\exists y, \beta \rightarrow \forall u,\ \exists v,\ \gamma)]$

$\neg[\forall x,\ \alpha \rightarrow (\forall y,\ \neg\beta \vee \forall u,\ \exists v,\ \gamma)]$

$\neg[\exists x,\ \neg\alpha \vee (\forall y,\ \neg\beta \vee \forall u,\ \exists v,\ \gamma)]$

$[\forall x, \alpha \wedge (\exists y, \beta \wedge (\exists u, \forall v, \neg\gamma))]$

Which is option (d).

1.28 (b)

1. $P \vee \sim Q \equiv P + Q'$
2. $\sim(\sim P \wedge Q) \equiv (P'\ Q)' \equiv P + Q'$
3. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\equiv PQ + PQ' + P'Q'$
$\equiv P(Q + Q') + P'Q'$
$\equiv P + P'Q'$
$\equiv (P + P')\ (P + Q') \equiv P + Q'$

4. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

$$\equiv PQ + PQ' + P'Q$$
$$\equiv P(Q + Q') + P'Q \equiv P + P'Q$$
$$\equiv (P + P')(P + Q) = P + Q$$

Clearly (i), (ii) and (iii) are equivalent. Correct choice is (b).

1.29 (d)

The correct translation of "Gold and silver ornaments are precious" is choice (d)

$$\forall x\,((G(x) \vee S(x)) \rightarrow P(x))$$

which is read as "if an ornament is gold or silver, then it is precious".

Now since a given ornament cannot be both gold and silver at the same time.

Choice (b) $\forall x\,((G(x) \wedge S(x)) \rightarrow P(x))$ is incorrect.

1.30 (b)

The given table can be converted into boolean function by adding minterms corresponding to true rows.

$\therefore$

P	Q	$P \square Q$
T	T	T
T	F	T
F	T	F
F	F	T

Since there is only one false in the above truth table, we can represent the function $P \square Q$ more efficientely, in conjunctive normal form.

Translates $P \square Q = P + Q'$ (the max-term corresponding to the third row, where the function is false).

Now, we can easily translate the choices into boolean algebra as follows:

Choice (a) $\neg Q \square \neg P \equiv Q' \square P' \equiv Q' + P$

Choice (b) $P \square \neg Q \equiv P \square Q' \equiv P + Q$

Choice (c) $\neg P \square Q \equiv P' \square Q \equiv P' + Q'$

Choice (d) $\neg P \square \neg Q \equiv P' \square Q' \equiv P' + Q$

As we can clearly see only choice (b) $P \square \neg Q$ is equivalent to $P + Q$.

1.31 (b)

I $\quad \neg \forall x P(x) \equiv \exists x \neg P(x)$

and IV $\quad \exists x \neg P(x)$

Clearly, choices I and IV are equivalent.

II $\quad \neg \exists x P(x) \equiv \forall x \neg P(x)$

and III $\quad \neg \exists x [\neg P(x)] \equiv \forall x P(x)$

Clearly II and III are not equivalent to each other or to I and IV.

1.32 (b)

$\forall x\, \exists y\, \exists t\, \neg F(x, y, t)$

$\equiv \neg\{\exists x\, \forall y\, \forall t\, F(x, y, t)\}$

$\equiv$ it is not true that (someone can fool all people at all time)

$\equiv$ no one can fool everyone all the time

1.33 (a)

If $P(x)$ is true, then

$x \neq 1$ and also

x is broken into two factors, only if, one of the factors is x itself and the other factor is 1, which is exactly the definition of a prime number.

So $P(x)$ is true means x is a prime number.

1.34 (b)

Let p : It rains

q : cricket match will not be played.

$$I_1: \quad \begin{array}{l} p \Rightarrow q \\ \underline{\sim q} \\ \therefore\ \sim p \end{array}$$

Clearly I_1 is correct since it is in the form of Modus Tollens (rule of contrapositive)

$$I_2: \quad \begin{array}{l} p \Rightarrow q \\ \underline{\sim p} \\ \therefore\ \underline{\sim q} \end{array}$$

which corresponds $[p \Rightarrow q \wedge \sim p] \Rightarrow \sim q$

$$\equiv [(p' + q)\, p'] \Rightarrow q'$$
$$\equiv [p' + qp'] \Rightarrow q' \equiv p' \Rightarrow q'$$
$$\equiv (p')' + q' \equiv p + q'$$

which is not a tautology.

So I_2 is incorrect inference.

1.35 (c)

Some real numbers are rational

$\equiv \exists x\, [\text{real}(x) \wedge \text{rational}(x)]$

1.36 (d)

None of my friends are perfect i.e., all of my friends are not perfect

$\forall x((F(x) \rightarrow \neg P(x))$

$\forall x(\neg F(x) \vee \neg P(x))$

$\neg \exists x\, (F(x) \wedge P(x))$

Alternatively Method:

$\exists x\, (F(x) \wedge P(x))$ gives

there exist some of my friends who are perfect.

$\neg \exists x\, (F(x) \wedge P(x))$

there does not exits any friend who is perfect i.e., none of my friends are perfect.

So (d) is correct option.